Exercise 8.3

1. Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$ (ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$ (iii) $\cos 48^\circ - \sin 42^\circ$ (iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Sol. (i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

$\because \sin(90^\circ - A) = \cos A$

And $\sin 18^\circ = \sin(90^\circ - 72^\circ)$

$\because \sin(90^\circ - 72^\circ) = \cos 72^\circ$

$\Rightarrow \sin 18^\circ = \cos 72^\circ$

$\therefore \frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

$\Rightarrow \frac{\sin 18^\circ}{\cos 72^\circ} = 1$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

We have $\tan 26^\circ = (\tan 90^\circ - 64^\circ) = \cot 64^\circ$ $[\because \tan(90^\circ - A) = \cot A]$

$\therefore \frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

$\Rightarrow \frac{\tan 26^\circ}{\cot 64^\circ} = 1$

(iii) $\cos 48^\circ - \sin 42^\circ$

$\because \cos 48^\circ - \sin(90^\circ - 42^\circ) = \sin 42^\circ$ $[\because \cos(90^\circ - A) = \sin A]$

$\therefore \cos 48^\circ - \sin 42^\circ = \sin 42^\circ - \sin 42^\circ$

$= 0$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

$\operatorname{cosec} 31^\circ - \sec(90^\circ - 59^\circ) = \sec 59^\circ$ $[\because \operatorname{cosec}(90^\circ - A) = \sin A]$

$\operatorname{cosec} 31^\circ - \sec 59^\circ = \sec 59^\circ - \sec 59^\circ$

$= 0.$

2. Show that:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Sol. (i) tan 48° tan 23° tan 42° tan 67° =1

L.H.S. = tan 48° tan 23° tan 42° tan 67°

= tan (90° - 42°) tan 23° tan.42°.tan (90.° - 23°)

= cot 42° tan 23° tan 42° cot 23° [tan (90 - A) = cot A]

$= \frac{1}{\tan 42^\circ} \times \tan 23^\circ \times \tan 42^\circ \times \frac{1}{\tan 23^\circ}$ $\left[\because \cot A = \frac{1}{\tan A}\right]$

$= \frac{\tan 42^\circ}{\tan 42^\circ} \times \frac{\tan 23^\circ}{\tan 23^\circ} = 1$

= R.H.S.

Thus, tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52°

L.H.S. = cos 38° cos 52° – sin 52°

= cos 38° cos (90°– 38°)- sin 38° sin (90° – 38°)

= cos(38° sin 38° – sin 38° cos 38°

[∵sin (90° – A) = cos A and cos(90° – A) = sin A]

= 0 = R.H.S.

This, cos 38° cos 52° – sin 38° sin 52° = 0

3. If tan 2A =cot (A - 18^0), where 2A is an acute angle, find the value of A.

Sol. Since tan 2A = cot (A –18°)

Also tan (2A)° = cot (90° – 2A) [∵tan θ = cot (90° – θ)]

⇒ A – 18 = 90°– 2A

⇒ A + 2A = 90° + 18°

⇒ 3A = 108°

$\Rightarrow A = \frac{108}{3} = 36^\circ.$

4. If tan A = cot B, prove that A + B = 90°.

Sol. tan A = cot B (given)

And cot B = tan (90° – B) [∵tan (90° – θ) = cot θ)]

∴ A = 90° – B

∴ A + B = 90°.

5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.

Sol. sec 4A = cosec (A – 20°)

sec (4A) = cosec (90° – 4A) [∵cosec (90° – θ) = sec θ)]

$\therefore$ A – 20° = 90° – 4A

$\Rightarrow$A + 4A = 90° + 20°

$\Rightarrow$ 5A = 110°

$$\Rightarrow A = \frac{110}{5} = 22°.$$

6. If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}.$$

Sol. Since, sum of the angles of ΔABC is A° + B° + C° = 180°

$\therefore$ B + C = 180° – A

Dividing both sides by 2,

$$\frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\therefore \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2} \qquad [\because \sin(90° - \theta) = \cos\theta]$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}.$$

7. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Sol. Since sin 67° = sin(90° – 23°)

= cos 23° [∵sin (90° – θ) = cos θ]

Also, cos 75° = cos (90° – 15) [∵cos(90° – θ) = sin θ]

= sin 15°

$\therefore$ We have:

sin 67° + cos 75° = cos 23° + sin 15°.